Investing money

8

syllabus reference

Financial mathematics 2
- **Investing money**

In this chapter

Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest Preliminary Course* CD-ROM or ask your teacher for a copy.

8.1 Converting percentages to decimals

1 Convert each of the following percentages to decimals.

a 12% **b** 8% **c** 4.5% **d** $6\frac{1}{4}\%$

8.2 Substitution into formulae

2 Given:

a $A = \frac{1}{2}bh$, find A when $b = 6.8$ and $h = 18.2$

b $T = a + (n - 1)d$, find T when $a = 7$, $n = 19$ and $d = 5$

c $I = Prn$, find I when $P = 12\,000$, $r = 0.065$ and $n = 7$.

8.3 Graphing linear and non-linear relationships

3 Use the data in the table to draw a graph of the relationship between population and time.

Time	0	1	2	3
Population	10 000	12 000	14 400	17 280

8.4 Converting units of time

4 Convert:

a 0 months into years

b 15 months into years

c 78 weeks into years.

8.5 Finding a percentage of a quantity

5 Find:

a 6% of \$1200 **b** 8.5% of \$2350 **c** 0.6% of \$12 500.

8.6 Writing one quantity as a percentage of another

6 Write:

a \$56 as a percentage of \$800

b \$1050 as a percentage of \$2800

c \$625 as a percentage of \$250 000.

Calculation of simple interest

When you deposit money in a bank, building society, or other financial institution you are actually lending them your money. Since you are lending them money you expect to receive your money back, plus an extra amount commonly known as **interest**. Similarly, if you borrow money from an institution, you must pay back the original sum, together with interest.

A measure of the interest paid is called the **interest rate**. The interest rate is a percentage of the amount of money invested or borrowed and is paid each year. Even though all interest rates are expressed in the same way, interest can be calculated by using several different methods. Whether depositing or borrowing, it is important that you understand how the interest is calculated.

The simplest method of interest calculation is called **simple interest**. Interest is calculated as a percentage of the initial deposit or borrowing (called the **principal**) and multiplied by the period the money was invested.

The formula used to calculate simple interest is:

$$I = Prn$$

where: I = simple interest
P = initial quantity
r = percentage interest rate per period, expressed as a decimal
n = number of periods

WORKED Example 1

Calculate the simple interest earned on an investment of $5000 at 4% p.a. for 3 years.

THINK	WRITE
1 Write down the simple interest formula.	$I = Prn$
2 Write down the values of P, r (converting the percentage to a decimal) and n.	$P = \$5000$ $r = 0.04$ $n = 3$
3 Substitute into the formula.	$I = \$5000 \times 0.04 \times 3$
4 Calculate.	$= \$600$

The total amount (A) that your deposit or debt has become after interest is added can be found using the formula:

$$A = P + I$$

where: A = total amount at the end of the term
P = initial quantity
I = simple interest

WORKED Example 2

$12 000 is invested for 5 years at 9.5% p.a. simple interest. Calculate the value of this investment at the end of the term.

THINK	WRITE
1 Write down the formula for simple interest.	$I = Prn$
2 Write down the value of P, r and n.	$P = \$12\,000$, $r = 0.095$, $n = 5$
3 Substitute the values into the given formula.	$I = \$12\,000 \times 0.095 \times 5$
4 Calculate the simple interest.	$= \$5700$
5 Write down the formula for the total amount.	$A = P + I$
6 Substitute the values into the given formula.	$= \$12\,000 + \$5700$
7 Calculate.	$= \$17\,700$

Graphics Calculator tip! Calculating simple interest

Your Casio graphics calculator can perform a number of financial functions by using the TVM mode. One of the options in this mode is to calculate simple interest. Examples such as worked example 2 are solved more simply by using the method above, rather than using the TVM mode of your graphics calculator. However, for some of the more complex questions later in this chapter, it is worth familiarising yourself with this method.

1. From the MENU select TVM.

2. Press F1 to select Simple Interest.

3. The calculator has two modes of calculating interest: 360 day mode or 365 day mode. You need to make sure that it is on 365 day mode. If not, press SHIFT SET UP, highlight DATE MODE and press F1 for 365.

4. Press EXE to return to the previous screen and enter the data for worked example 2.
   n = 3 × 365 (as n is in days)
   I% = 4
   PV = –5000
   (Principal or present value is entered as a negative.)

5. The calculator gives you two options.
   F1: SI is simple interest.
   F2: SFV is future value, in other other words the principal plus interest.
   In this example, as we want the simple interest, we press F1 for SI.

Care must be taken with simple interest questions when the length of the investment is not given in years. If the investment is given as months, it must be converted to years by writing the number of months as a fraction over 12, for example,

$$18 \text{ months} = \frac{18}{12} = 1\frac{1}{2} \text{ years.}$$

WORKED Example 3

Calculate the simple interest earned on an investment of $7600 at 5.2% p.a. for 9 months.

THINK	WRITE
1 Write down the formula for simple interest.	$I = Prn$
2 Write down the value of P, r and n (converting 9 months to $\frac{9}{12} = \frac{3}{4}$ year).	$P = \$7600$, $r = 0.052$, $n = \frac{3}{4}$
3 Substitute the values into the given formula.	$I = \$7600 \times 0.052 \times \frac{3}{4}$
4 Calculate the simple interest.	$= \$296.40$

Some examples will ask you to calculate the length of time for which money must be invested in order to earn a given amount of interest.

WORKED Example 4

How long, to the nearest month, will it take to earn $650 simple interest if $8375 is invested at 6.25% p.a.?

THINK	WRITE
1 Write down the formula for simple interest.	$I = Prn$
2 Write down the value of I, P and r.	$I = \$650$, $P = \$8375$, $r = 0.0625$

Continued over page

THINK	WRITE
3 Substitute the values into the given formula.	$\$650 = \$8375 \times 0.0625 \times n$
4 Simplify the RHS of the equation.	$= \$523.4375 \times n$
5 Make n the subject of the equation.	$n = \frac{\$650}{\$523.4375}$
6 Calculate the value of n in years.	$= 1.2418$ years
7 Convert 0.2418 years to months by multiplying the decimal by 12.	≈ 15 months
8 Answer the question.	It will take approximately 1 year and 3 months to earn \$650 in simple interest.

Examples of investments involving simple interest include investment bonds and debentures. **Investment bonds** are offered by the government, either State or federal, and larger organisations such as Telstra.

Interest earned on investment bonds can be paid at varying intervals, for example monthly, quarterly, every six months (semi-annually) or yearly. Bonds are traded on financial markets. That is, they can be bought or sold prior to the term expiry date (also known as bond maturity).

Debentures are similar to investment bonds but are issued by private companies to investors to raise capital. At the end of the term, the principal (or face value) is returned to the investor, while the interest earned is again paid at varying intervals.

remember

1. Interest is an amount of money paid to an investor or by a borrower for the use of money.
2. Simple interest is calculated by using only the initial investment.
3. The simple interest formula is:
$$I = Prn \quad \text{FS}$$
where P is the initial quantity, r is the interest rate per period expressed as a decimal and n is the number of periods.
4. If using the formula to find a value other than I, substitute the known values into the formula and solve the resulting equation.

EXERCISE 8A Calculation of simple interest

SKILLSHEET 8.1 Converting percentages to decimals

SKILLSHEET 8.2 Substitution into formula

WORKED Example 1

1 Veronica invests \$4000 for 3 years at 5% p.a. Calculate the simple interest earned.

2 In each of the following, calculate the amount of simple interest earned.
- a \$1200 for 1 year at 10% p.a.
- b \$2460 for 5 years at 5% p.a.
- c \$126 000 for 2 years at 8.5% p.a.
- d \$9862 for 6 years at 11.25% p.a.
- e \$1000 for $1\frac{1}{2}$ years at 6% p.a.
- f \$1750 for $5\frac{1}{4}$ years at 7.45% p.a.

3 Julie has $40 000 to invest. She invests it for 5 years in a State Government bond that pays 5.6% p.a. interest. Calculate the simple interest that Julie will earn:
- **a** each year
- **b** for the whole 5 years of the investment.

WORKED Example 2

4 Brian has a $10 000 inheritance that he wants to invest. He invests his money in government bonds for 3 years at 8% p.a. Calculate:
- **a** the simple interest earned
- **b** the value of his investment on maturity.

5 Karelle invests $7600 in a debenture that pays 6.9% p.a. for investments over 2 years. Calculate the total value of Karelle's investment on maturity.

6 Frank is 7 years old and starts a savings account with the local bank. He has $140 with which to start the account.
- **a** If the interest rate is $3\frac{3}{4}\%$, calculate the amount of interest Frank will receive after one year.
- **b** What will be the balance of Frank's bank account after one year?

7 Loretta invests $7540 at 5.95% p.a.
- **a** Calculate the simple interest that Loretta will earn in her first year.
- **b** Loretta receives two interest payments per year. Calculate the size of each payment.
- **c** Find the total value of the investment after $4\frac{1}{2}$ years.

WORKED Example 3

8 Kath invests $9450 in a government bond that pays 6% p.a. simple interest for an 18 month investment. Calculate the simple interest earned on this investment.

9 **multiple choice**

The simple interest paid on an investment of $5750 at 4.6% p.a. for 2 years is:

A $529.00 **B** $541.17 **C** $6291.17 **D** $6279.00

10 **multiple choice**

The total value of an investment of $3500 after 2 years and 6 months if simple interest is paid at the rate of 5% per annum, is:

A $437.50 **B** $826.25 **C** $3937.50 **D** $3975.50

11 Calculate the simple interest that has to be paid, if $4650 is invested on a term deposit for 180 days at 5.75% p.a. (*Hint:* Write 180 days as a fraction of 1 year.)

WORKED Example 4

12 How long, to the nearest month, will it take to earn $2400 simple interest, if $16 410 is invested at 9.75% p.a.?

13 A debenture offers to pay 8% p.a. interest on a 4 year investment. Janine wants to earn $2000 interest. What principal will Janine need to invest?

14 **multiple choice**

What sum, to the nearest dollar, must be invested for one year at 6% per annum simple interest, in order to earn $1200 interest?

A $2000 **B** $12 200 **C** $20 000 **D** $21 200

15 Sue and Harry invested $14 500 in State Government bonds at 8.65% p.a. The investment is for 10 years and the interest is paid semi-annually (that is, every six months). Calculate how much interest:
- **a** they receive every payment
- **b** will be received in total.

16 Mrs Williams invested $60 000 in government bonds at 7.5% p.a. with interest paid semi-annually (that is, every 6 months).
- **a** How much interest is she paid each 6 months?
- **b** How much interest is she paid over 3 years?
- **c** How long would the money need to be invested to earn a total of $33 750 in interest?

17 Mr and Mrs Tyquin donate money for a scholarship at the local high school. The value of the scholarship is $1500. They invest a sum of money at 8% p.a. so that each year $1500 in interest is earned. How much will Mr and Mrs Tyquin need to invest?

Find the simple interest on each of the following investments.

1 $4000 at 5% p.a. for 4 years

2 $9000 at 7% p.a. for 2 years

3 $15 000 at 6% p.a. for 3 years

4 $950 at 0.1% p.a. for 2 years

5 $40 000 at 3.5% p.a. for 5 years

6 $1200 at 4.6% p.a. for $2\frac{1}{2}$ years

7 $5745 at $3\frac{3}{4}$% p.a. for 1 year

8 $32 500 at 4.1% p.a. for 18 months

9 $532 at 0.2% p.a. for 6 months

10 $3330 at 6.95% p.a. for 9 months

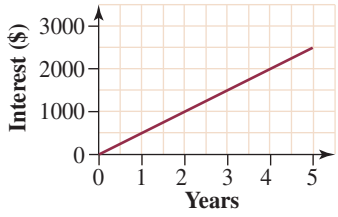
Graphing simple interest functions

Suppose that we invest $10 000 at 5% p.a. simple interest. The table below shows the amount of interest that we will receive over various lengths of time.

No. of years	1	2	3	4	5
Interest	$500	$1000	$1500	$2000	$2500

The amount of interest earned can be graphed by the linear function at right.

Note that the gradient of this graph is 500, which is the amount of one year's interest, or 5% of the principal.



WORKED Example 5

$6000 is invested at 4% p.a.

a Complete the table below to calculate the interest that will have been earned over 5 years.

No. of years	1	2	3	4	5
Interest					

b Graph the interest earned against the number of years the money is invested.

THINK

a Use the simple interest formula to calculate the interest earned on $6000 at 4% p.a. for 1, 2, 3, 4 and 5 years.

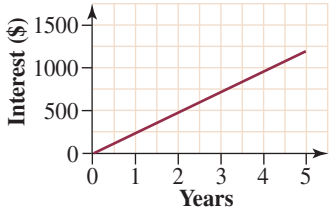
b Draw the graph with Years on the horizontal axis and Interest on the vertical axis.

WRITE/DRAW

a

No. of years	1	2	3	4	5
Interest	$240	$480	$720	$960	$1200

b

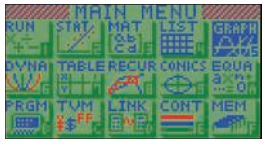


Graphics Calculator tip! Graphing a simple interest function

By using the simple interest formula we can create a function that can be graphed using the GRAPH function on the graphics calculator.

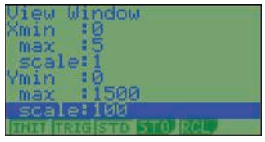
Consider worked example 5.

1. Write the simple interest formula $I = Prn$
2. Substitute the known values of $P = \$6000$ and $r = 0.04$. Simplify the expression. $I = 6000 \times 0.04 \times n$ $= 240n$
3. From the **MENU** select **GRAPH**.

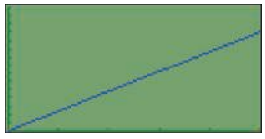


4. Delete any existing function and enter **Y1 = 240X**.

5. To draw up the axes press **SHIFT** **F3** for **V-Window** and enter the setting shown at right.



6. Press **EXE** to return to the previous screen and then press **F6** to **DRAW** the graph.



We are able to compare the interest that is earned by an investment at varying interest rates by graphing the interest earned at varying rates on the one set of axes.

WORKED Example 6

Kylie has $12 000 to invest. Three different banks offer interest rates of 4%, 5% and 6%.

a Complete the table below to show the interest that she would earn over 5 years.

No. of years	1	2	3	4	5
Interest (4%)					
Interest (5%)					
Interest (6%)					

b Show this information in graph form.

THINK

a 1 Use the simple interest formula to calculate the interest earned on $12 000 at 4% p.a. for 1, 2, 3, 4 and 5 years.

2 Use the simple interest formula to calculate the interest earned on $12 000 at 5% p.a. for 1, 2, 3, 4 and 5 years.

3 Use the simple interest formula to calculate the interest earned on $12 000 at 6% p.a. for 1, 2, 3, 4 and 5 years.

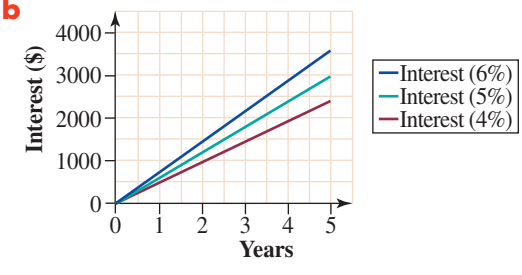
b Draw a line graph for each investment.

WRITE/DRAW

a

No. of years	1	2	3	4	5
Interest (4%)	$480	$960	$1440	$1920	$2400
Interest (5%)	$600	$1200	$1800	$2400	$3000
Interest (6%)	$720	$1440	$2160	$2880	$3600

b



remember

1. The amount of simple interest earned on an investment for various lengths of time can be graphed as a linear function.
2. The gradient of the linear function will be equal to one year's interest.
3. To compare the interest earned at various rates, we can graph several functions on the one set of axes.

EXERCISE 8B Graphing simple interest functions

WORKED Example 5

1 $8000 is invested at 5% p.a.

a Copy and complete the table below to calculate the interest over 5 years.

No. of years	1	2	3	4	5
Interest					

b Draw a graph of the interest earned against the length of the investment.

8.3 Skillsheet Graphing linear and non-linear relationships

2 $20 000 is to be invested at 8% p.a.

a Copy and complete the table below to calculate the interest for various lengths of time.

No. of years	1	2	3	4	5
Interest					

b Draw a graph of the interest earned against the length of the investment.

c What is the gradient of the linear graph drawn?

d Use your graph to find the amount of interest that would have been earned after 10 years.

3 Draw a graph to represent the interest earned by each of the following investments over 5 years.

a $15 000 at 7% p.a.

b $2000 at 10% p.a.

c $8600 at 7.5% p.a.

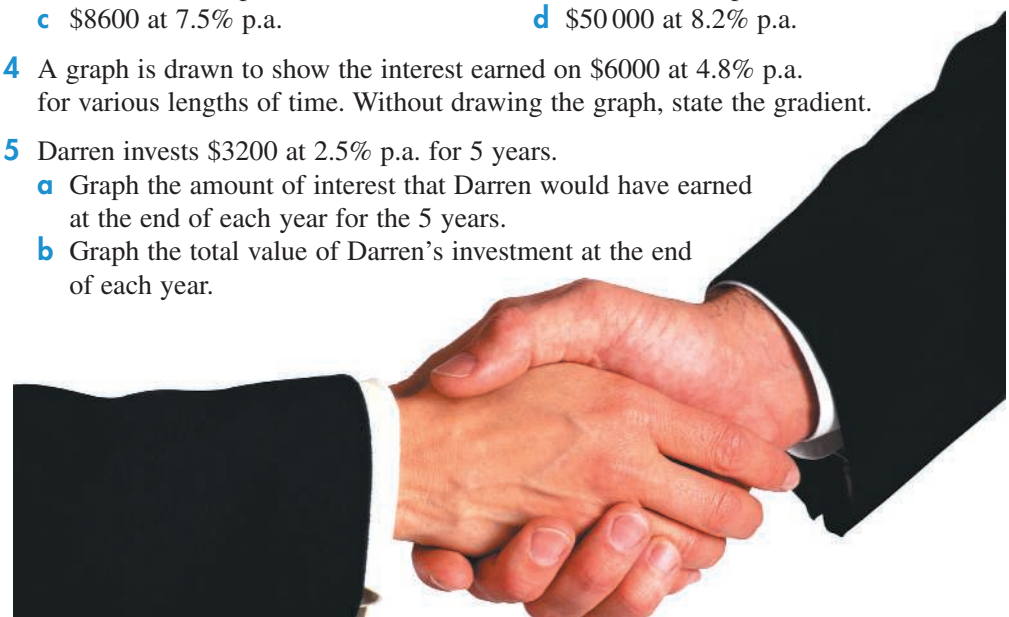
d $50 000 at 8.2% p.a.

4 A graph is drawn to show the interest earned on $6000 at 4.8% p.a. for various lengths of time. Without drawing the graph, state the gradient.

5 Darren invests $3200 at 2.5% p.a. for 5 years.

a Graph the amount of interest that Darren would have earned at the end of each year for the 5 years.

b Graph the total value of Darren's investment at the end of each year.



WORKED Example 6

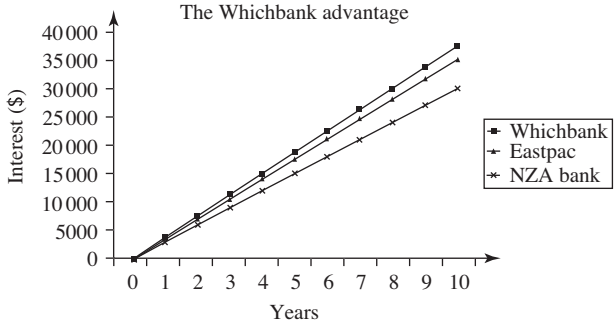
6 Julieanne has $25 000 to invest at 5%, 6% or 8%.

a Complete the table below to show the interest that she would earn over 5 years.

No. of years	1	2	3	4	5
Interest (5%)					
Interest (6%)					
Interest (8%)					

b Show this information in graph form.

7 Theo has $50 000 to invest. Theo investigates the website www.whichbank.com.au, which has an interactive component. Theo enters the figure $50 000 and the following graph is displayed.



a Find the amount of simple interest earned after 10 years by investing with each of the three banks listed.

b Use your answer to **a** to calculate the interest rate paid by each of the three banks.

8 Mark has $5500 to invest at 3%, 3.5% or 3.75%.

a Complete the table below to show the interest that he would earn over various lengths of time.

No. of years	1	2	3	4	5
Interest (3%)					
Interest (3.5%)					
Interest (3.75%)					

b Show this information in graph form.

9 Draw a graph to show the interest earned on an investment of $12 500 at 4.5% p.a., 5% p.a. and 5.2% p.a. Use the graph to find:

a the amount of interest earned by each investment after 8 years

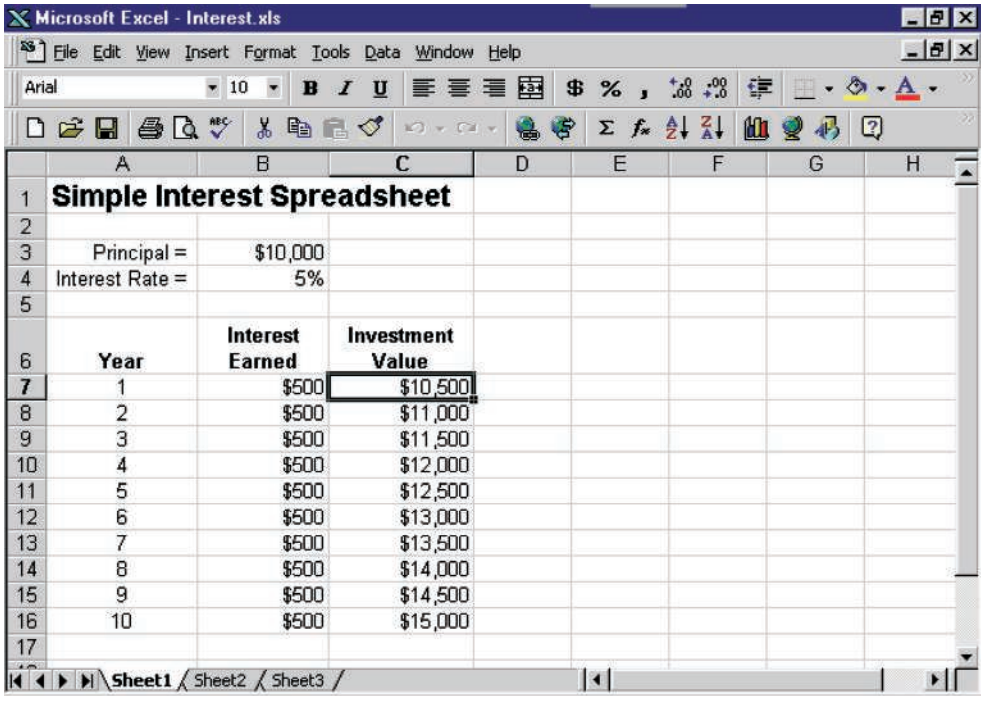
b how much more the investment at 5.2% p.a. is worth after 10 years than the 4.5% p.a. investment.

10 Three banks offer $4000 debentures at rates of 5.2% p.a., 5.8% p.a. and 6.2% p.a. Draw a graph of the value of the debentures at maturity against the number of years of the debenture.

Computer Application 1 Simple interest spreadsheets

Throughout this chapter we will use some spreadsheets that allow us to track the growing value of an investment over time.

1. From the *Maths Quest General Mathematics Preliminary Course* CD-ROM open the spreadsheet 'Interest'.



2. The spreadsheet 'Simple Interest' (Sheet 1) models an investment of $10 000 at 5% p.a.
3. Use the graphing function on your spreadsheet to draw a line graph for the amount of interest earned each year and the value of the investment after each year.
4. Change the amount of the principal and the interest rate, and note the change in the figures displayed and the chart.
5. Use this function to check your answers to Exercise 8B.
6. Save the spreadsheet as Simple Interest.

Calculation of compound interest

In practice, most investments are not calculated using simple interest. If you have a bank account, you would know that when interest is paid the balance of your account grows and it is on this new balance that your next interest payment is calculated.

When interest is added to the principal and this new balance is used to calculate the next interest payment, this is called **compound interest**. We can calculate compound interest by calculating simple interest one period at a time. The amount to which the initial investment grows is called the **compounded value** or **future value**.

WORKED Example 7

Calculate the future value of an investment of $10 000 at 10% p.a. for 3 years with interest paid at the end of each year, by calculating the simple interest for each year separately.

THINK	WRITE
1 Write the initial principal.	Initial principal = $10 000
2 Calculate the interest for the 1st year.	1st year's interest = 10% of $10 000 = $1000
3 Calculate the 2nd year's principal by adding the 1st year's interest to the initial principal.	2nd year's principal = $10 000 + $1000 = $11 000
4 Calculate the 2nd year's interest.	2nd year's interest = 10% of $11 000 = $1100
5 Calculate the 3rd year's principal by adding the 2nd year's interest to the 2nd year's principal.	3rd year's principal = $11 000 + $1100 = $12 100
6 Calculate the 3rd year's interest.	3rd year's interest = 10% of $12 100 = $1210
7 Calculate the future value of the investment by adding the 3rd year's interest to the 3rd year's principal.	Future value = $12 100 + $1210 = $13 310

To calculate the actual amount of interest received, we subtract the initial principal from the future value.

In the example above $CI = \$13\,310 - \$10\,000$
$= \$3310$

To compare this with simple interest earnings at the same rate.

$I = Prn$
$= \$10\,000 \times 0.1 \times 3$
$= \$3000$

The table below shows a comparison between the value of an investment of $10 000 earning 10% p.a. at both simple interest and compound interest.

Year	1	2	3	4	5	6	7	8
Simple interest	$1000	$2000	$3000	$4000	$5000	$6000	$7000	$8 000
Compound inteerest	$1000	$2100	$3310	$4641	$6105	$7716	$9487	$11 436

We can develop a formula for the future value of an investment rather than do each example by repeated use of simple interest. Consider worked example 7. Let the compounded value after each year be A_n.

After 1 year $A_1 = 10\,000 \times 1.1$ (increasing $10 000 by 10%)
After 2 years $A_2 = A_1 \times (1.1)$
$= 10\,000 \times 1.1 \times 1.1$ (substituting the value of A_1)
$= 10\,000 \times 1.1^2$

After 3 years $A_3 = A_2 \times 1.1$
$= 10\,000 \times 1.1^2 \times 1.1$
$= 10\,000 \times 1.1^3$

The pattern then continues such that the value of the investment after n years equals:

$$\$10\,000 \times 1.1^n$$

We can generalise this example to any investment.

$$A = P(1 + r)^n$$ FS

where A = final balance
P = initial quantity
r = percentage interest rate per compounding period, expressed as a decimal
n = number of compounding periods.

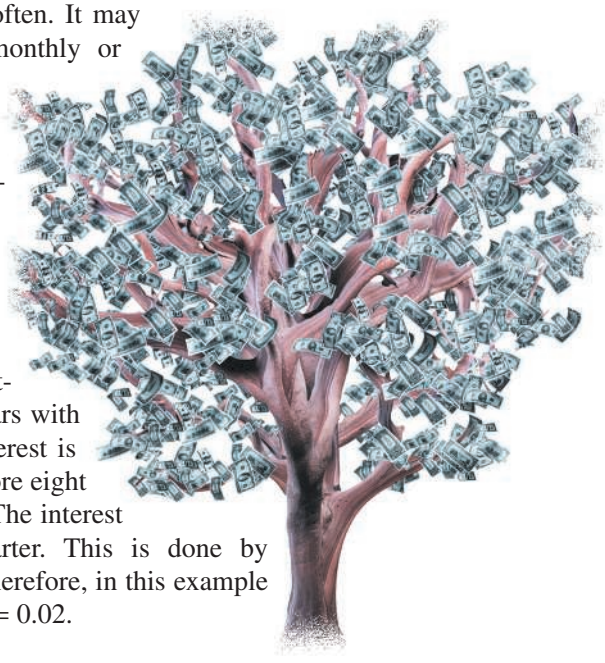
In the financial world, the terms future value (FV) and present value (PV) are sometimes used instead of amount and principal.

WORKED Example 8

Calculate the future value of an investment of $12 000 at 7% p.a. for 5 years, where interest is compounded annually.

THINK	WRITE
1 Write down the formula for the future value.	$A = P(1 + r)^n$
2 Write down the value of P, r (as a decimal) and n.	$P = \$12\,000$, $r = 0.07$, $n = 5$
3 Substitute into the formula.	$A = \$12\,000 \times 1.07^5$
4 Calculate.	$= \$16\,830.62$

In the above example, interest is paid annually; however, this is not always the case. In many cases interest is paid more often. It may be paid six-monthly, quarterly, monthly or even daily. This is called the **compounding period**. If interest is paid more often than annually, the value of n is the number of compounding periods during the investment. The interest rate then needs to be converted from a rate per annum to a rate per compounding period.



For example, consider an investment of $6000 at 8% p.a. for 2 years with interest compounded quarterly. Interest is paid four times per year and therefore eight times in 2 years. Therefore $n = 8$. The interest rate must be calculated per quarter. This is done by dividing the annual rate by four. Therefore, in this example the rate is 2% per quarter, hence $r = 0.02$.

WORKED Example 9

Calculate the future value of an investment of $6000 at 8% p.a. for 2 years with interest compounded quarterly.

THINK	WRITE
1 Write down the formula for the future value.	$A = P(1 + r)^n$
2 Write down the value of P, r (as a decimal) and n.	$P = \$6000$, $r = 0.02$, $n = 8$
3 Substitute into the formula.	$A = \$6000 \times 1.02^8$
4 Calculate.	$= \$7029.96$

Graphics Calculator tip! Compound interest warning!

The TVM function on a graphics calculator can calculate compound interest. This is somewhat more complicated than using the formula $A = P(1 + r)^n$. Further, a graphics calculator uses an imprecise method and answers may vary by up to $1 from the correct answer. For this reason we recommend not using a graphics calculator for these calculations.

remember

1. The future value of an investment under compound interest can be calculated by calculating the simple interest for each year separately.
2. The formula used to calculate the future value of an investment is:
$$A = P(1 + r)^n$$ FS
where A = final balance
P = initial quantity
n = number of compounding periods
r = percentage interest rate per compounding period, expressed as a decimal.
3. In the formula, n is the number of compounding periods over the term of the investment and r is the interest rate per compounding period. To find r, divide the interest rate per annum by the number of compounding periods per year.

EXERCISE 8C Calculation of compound interest

SkillSHEET 8.4 Converting units of time

WORKED Example 7

1 Ray has $5000 to invest. He invests it for 3 years at 10% p.a. with interest paid annually. Calculate the future value of the investment by calculating the simple interest on each year separately.

2 Suzanne is to invest $15 000 for 2 years at 7% p.a. with interest paid annually. Calculate the future value of the investment by calculating the simple interest for each year separately.

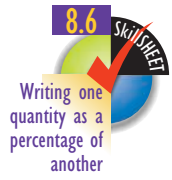
SkillSHEET 8.5 Finding a percentage of a quantity

WORKED Example 8

3 Kiri has $2000 to invest. She invests the money at 8% p.a. for 5 years with interest compounded annually. Use the formula $A = P(1 + r)^n$ to calculate the future value of Kiri's investment.

4 Use the compound interest formula to calculate the future value of each of the following investments with interest compounded annually.

a $4000 at 5% p.a. for 3 years
b $8000 at 3% p.a. for 5 years
c $18 000 at 8% p.a. for 4 years
d $11 500 at 5.5% p.a. for 3 years
e $8750 at 6.25% p.a. for 6 years



WORKED Example 9

5 Carla is to invest $45 000 at 9.2% p.a. for 5 years with interest compounded six-monthly. Calculate the future value of the investment.

6 A passbook savings account pays interest of 0.2% p.a. Luke has $500 in such an account. Calculate the future value of the account after 2 years, if interest is compounded quarterly.

7 Noel is to invest $12 000 at 8% p.a. for 2 years with interest compounded quarterly. Calculate the future value of the investment.

8 Vicky invests $30 000 in a one-year fixed deposit at an interest rate of 6% p.a. with interest compounding monthly.

a Convert the interest rate of 6% p.a. to a rate per month.
b Calculate the future value of the investment upon maturity.

9 Calculate the compounded value of each of the following investments.

a $960 for 1 year at 4.50% p.a. with interest compounded six-monthly.
b $7500 for $3\frac{1}{2}$ years at 5.6% p.a. with interest compounded quarterly.
c $152 000 for $2\frac{1}{2}$ years at 7.2% p.a. with interest compounded six-monthly.
d $14 000 for 4 years at 9% p.a. with interest compounded monthly.
e $120 000 for 20 years at 11.95% p.a. with interest compounded quarterly.

10 **multiple choice**

A sum of $5000 is invested for 2 years at the rate of 4.75% p.a., compounded quarterly. The interest paid on this investment, to the nearest dollar, is:

A $475 **B** $495 **C** $5475 **D** $5495

11 **multiple choice**

After selling their house Mr and Mrs Dengate have $61 800. They plan to invest it at 6% p.a., with interest compounded annually. The value of their investment will first exceed $100 000 after:

A 8 years **B** 9 years **C** 10 years **D** 11 years

12 **multiple choice**

Warren wishes to invest $10 000 for a period of 5 years. The following investment alternatives are suggested to him. The best investment would be:

A simple interest at 9% p.a.
B compound interest at 8% p.a. with interest compounded annually
C compound interest at 7.8% p.a. with interest compounded six-monthly
D compound interest at 7.2% p.a. with interest compounded quarterly

13 **multiple choice**

Which of the following investments, to be invested for 6 years and compounded semi-annually at 8% p.a., will have a future value closest to $15 000?

A $900 **B** $8500 **C** $9400 **D** $11 000

14 Brittany has $13 500 to invest. An investment over a 2-year term will pay interest of 8% p.a. Calculate the compounded value of Brittany's investment if the compounding period is:
a one year **b** six months **c** three months **d** monthly.

15 Kerry invests $100 000 at 8% p.a. for a one-year term. For such large investments interest is compounded daily.
a Calculate the daily percentage interest rate, correct to 4 decimal places.
b Calculate the compounded value of Kerry's investment on maturity.
c Calculate the amount of interest paid on this investment.
d Calculate the extra amount of interest earned, compared with the interest calculated at the end of the year.

16 Simon invests $4000 for 3 years at 6% p.a. simple interest. Monica also invests $4000 for 3 years, but her interest rate is 5.6% p.a. with interest compounded quarterly.
a Calculate the value of Simon's investment on maturity.
b Show that the compounded value of Monica's investment is greater than Simon's investment.
c Explain why Monica's investment is worth more than Simon's, despite receiving a lower rate of interest.

Computer Application 2 Compound interest spreadsheets

Earlier we wrote a spreadsheet to show the growth of an investment over a number of years. We will now write a similar spreadsheet to show the growth under compound interest.

1. From the *Maths Quest General Mathematics Preliminary Course* CD-ROM open the spreadsheet 'Interest'.

Microsoft Excel - Interest.xls

File Edit View Insert Format Tools Data Window Help

Arial 10

	A	B	C	D	E	F	G	H
1	**Compound Interest Spreadsheet**							
2								
3	Principal =	$10,000						
4	Interest Rate =	5%						
5	Compounding periods per year	1						
6	Interest rate per compounding period	5.00%						
7	**Year**	**Compounded Value**						
8	1	$10,500						
9	2	$11,025						
10	3	$11,576						
11	4	$12,155						
12	5	$12,763						
13	6	$13,401						
14	7	$14,071						
15	8	$14,775						
16	9	$15,513						
17	10	$16,289						
18								
19								

Sheet1 Sheet2 Sheet3

2. Select Sheet 2, 'Compound Interest'. This spreadsheet models a $10 000 investment at 5% p.a. interest with interest compounded annually (one compounding period per year).

3. Use the graphing function to draw a graph showing the growth of this investment over 10 years. Compare this graph with the graph drawn for the corresponding simple interest investment.
4. Change the number of compounding periods per year to see the change in the value of the investment. Your graph should change as you change the information.
5. Change other information, such as the principal and interest rate, to see the change in your graph.
6. Save this spreadsheet as Compound Interest.

10 QUICK QUESTIONS 2

1. Calculate the simple interest earned on an investment of $9240 made at 7.4% p.a. for 3 years.
2. Corey invests $14 200 for 5 years in debentures that pay 4.3% p.a. simple interest. Calculate the total value of Corey's investment at maturity.
3. Emma invests $27 500 in investment bonds which pay 6.25% p.a. simple interest for 2 years. Calculate the interest earned in this investment.
4. The interest that Emma receives is paid in quarterly instalments. Calculate the size of each quarterly interest payment.
5. Vladimir invests $2000 at 5% p.a. for 3 years with interest compounded annually. Calculate the compounded value of Vladimir's investment.
6. Calculate the amount of interest earned by Vladimir.
7. Calculate the compounded value of an investment of $6000 at 6.4% p.a. for 2 years with interest compounded annually.
8. Calculate the compounded value of an investment of $6000 at 6.4% p.a. for 2 years with interest compounded six-monthly.
9. Calculate the compounded value of an investment of $6000 at 6.4% p.a. for 2 years with interest compounded quarterly.
10. Calculate the compounded value of an investment of $13 200 at 7.2% p.a. for 18 months with interest compounded monthly.

Calculating compound interest from a table of compounded values

So far we have looked at the calculation of compounded values and the amount of compound interest paid. Suppose we have $10 000 saved for a world holiday, which is going to cost $15 000. The best interest rate for investing the money is at 8%, compounded quarterly. We want to know how long we need to invest the $10 000, so that it will have a compounded value of $15 000.

To solve this example we need to calculate the value of n, having been given the values of CV, PV and r. The best way to do this is to use a table showing the **compound value interest factor** for various investments.

A compound value interest factor (*CVIF*) is the compounded value that $1 will amount to under a certain investment. For example, if $1 were invested at 5% p.a. for 4 years, compounded annually, its compounded value would be $1.216. We can use this to calculate the value of other amounts of money under the same investment pattern.

For example, if $7600 were invested at 5% p.a. for 4 years, to calculate the compounded value of the investment we multiply $7600 by the *CVIF* which is 1.216.

Therefore $CV = \$7600 \times 1.216$
$= \$9241.60$

In this example $7600 is the **present value** (*PV*) of the investment and $9241.60 is the compounded value (*CV*).

We can therefore use the formula:

$$CV = PV \times CVIF$$

This formula simply states: compounded value = present value × interest factor.
The *CVIF* table below shows the interest factors.

	Interest rate per period									
Periods	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
1	1.010	1.020	1.030	1.040	1.050	1.060	1.070	1.080	1.090	1.100
2	1.020	1.040	1.061	1.082	1.103	1.124	1.145	1.166	1.188	1.210
3	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295	1.331
4	1.041	1.082	1.126	1.170	1.216	1.262	1.311	1.360	1.412	1.464
5	1.051	1.104	1.159	1.217	1.276	1.338	1.403	1.469	1.539	1.611
6	1.062	1.126	1.194	1.265	1.340	1.419	1.501	1.587	1.677	1.772
7	1.072	1.149	1.230	1.316	1.407	1.504	1.606	1.714	1.828	1.949
8	1.083	1.172	1.267	1.369	1.477	1.594	1.718	1.851	1.993	2.144
9	1.094	1.195	1.305	1.423	1.551	1.689	1.838	1.999	2.172	2.358
10	1.105	1.219	1.344	1.480	1.629	1.791	1.967	2.159	2.367	2.594

We can now use this table to solve compound interest problems.

WORKED Example 10

Use the *CVIF* table to find the compounded value of $4560 invested at 8% p.a. for 2 years with interest compounded six-monthly.

THINK	WRITE
1 Calculate the interest rate per period and number of interest periods.	Interest rate per period = 4% Interest periods = 4
2 Look up the *CVIF* for 4% with 4 interest periods.	*CVIF* = 1.170
3 Write the formula.	$CV = PV \times CVIF$
4 Substitute the *PV* and the *CVIF*.	$= \$4560 \times 1.170$
5 Calculate.	$= \$5335.20$

This table can also be used to help us calculate the present value of an investment that is required to produce a given compounded value.

This is done using the same formula; however, you will need to solve the equation to find the value of *PV*.

WORKED Example 11

Liz is 16 years old. She hopes to have $3000 in 3 years to buy a used car. She finds an investment of 6% p.a. with interest compounded six-monthly. Calculate the amount of money that Liz must invest to generate a compounded value of $3000 in 3 years.

THINK	WRITE
1 Calculate the interest rate per period and number of interest periods.	Interest rate per period = 3% Interest periods = 6
2 Look up the *CVIF* for 3% with 6 interest periods.	*CVIF* = 1.194
3 Write the formula.	$CV = PV \times CVIF$
4 Substitute for *CV* and *CVIF*.	$\$3000 = PV \times 1.194$
5 Make *PV* the subject of the equation (by dividing by 1.194).	$PV = \frac{\$3000}{1.194}$
6 Calculate.	$= \$2512.56$
7 Give a written answer.	Liz will need to invest $2512.56 to generate $3000 in 3 years.

We can also use the table to determine the length of time that a given present value will take to reach a certain compounded value. This is done by calculating the required *CVIF* and looking for the first *CVIF* in the table, at the given interest rate, greater than that required.

WORKED Example 12

How long will it take $2500 to grow to $3200 when invested at 8% p.a. with interest compounded six-monthly?

THINK

1. Calculate the interest rate per period.
2. Write the value of *PV* and *CV*.
3. Write the formula.
4. Substitute the values of *PV* and *CV*.
5. Make *CVIF* the subject of the formula.
6. Calculate the value of *CVIF*.
7. Look at the 4% column of the *CVIF* table. The first *CVIF* greater than 1.28 (that is, 1.316) will be the minimum number of interest periods required to produce the required growth.
8. Calculate the length of time for seven interest periods.

WRITE

Interest rate per period = 4%

$PV = \$2500$, $CV = \$3200$

$CV = PV \times CVIF$

$\$3200 = \$2500 \times CVIF$

$$CVIF = \frac{\$3200}{\$2500}$$

$$= 1.28$$

Seven interest periods will be required.

It will take $3\frac{1}{2}$ years for $2500 to grow to $3200.

The same method can be used to find the interest rate required to achieve a certain compounded value over a fixed period of time.

remember

1. The compounded value of $1 under a particular investment is called the compounded value interest factor (*CVIF*).
2. The compounded value of any investment can be calculated by using a *CVIF* table and the formula $CV = PV \times CVIF$.
3. We can use *CVIF* tables to calculate the length of time an investment will take to reach a certain compounded value, or the interest rate required to reach a certain compounded value.

EXERCISE 8D Calculating compound interest from a table of compounded values

WORKED Example 10

1 Toshika has $10 000 to invest for 4 years. The bank offers her 7% p.a. with interest compounded annually. Use the *CVIF* table on page 248 to calculate the compounded value of Toshika's investment.

2 Greg has $8500 to invest for 5 years. A building society offers 8% p.a. with interest compounded twice a year. Use the *CVIF* table to calculate the compounded value of Greg's investment.

3 Marlene invests $40 000 for 2 years at 8% p.a. with interest compounded quarterly. Use the *CVIF* table to calculate the compounded value of Marlene's investment.

4 Roger invests $2400 for 2 years in an 'at call' account, which pays 4% p.a. interest with interest paid quarterly. Use the *CVIF* table to calculate the future value of this investment.

5 Use the *CVIF* table to calculate the compounded value of each of the following investments.

a $5000 at 9% p.a. for 6 years with interest compounded annually
b $6700 at 10% p.a. for 4 years with interest compounded six-monthly
c $250 at 6% p.a. for 5 years with interest compounded six-monthly
d $23 670 at 4% p.a. for 2 years with interest compounded quarterly
e $13 250 at 8% p.a. for 18 months with interest compounded quarterly
f $115 000 at 12% p.a. for 6 months with interest compounded monthly

6 Use the formula $A = P(1 + r)^n$ to calculate the *CVIF,* correct to 3 decimal places, for an investment at 2.5% for:

a 1 interest period **b** 2 interest periods **c** 3 interest periods
d 4 interest periods **e** 6 interest periods **f** 8 interest periods.

7 Using the *CVIF*s found in question **6** will allow you to calculate each of the compounded values of the following investments.

a $900 at 2.5% p.a. for 3 years with interest compounded annually
b $2340 at 5% p.a. for 2 years with interest compounded six-monthly
c $7200 at 10% p.a. for 1 year with interest compounded quarterly
d $11 000 at 10% p.a. for 2 years with interest compounded quarterly
e $5750 at 10% p.a. for $1\frac{1}{2}$ years with interest compounded quarterly

8 multiple choice

One dollar invested at 3.5% for 5 interest periods amounts to:
A 0.175 **B** 1.035 **C** 1.175 **D** 1.188

9 multiple choice

For a certain investment the *CVIF* = 2.147. If the present value of the investment is $32 546, the compounded value, correct to the nearest dollar, will be:
A $15 158 **B** $15 159 **C** $69 876 **D** $69 877

WORKED Example 11

10 Jason wants to save for a car in 3 years. He needs to have $10 000. Use the *CVIF* table to calculate the amount of money that he will need to invest at 5% p.a. with interest compounded annually, to have $10 000 in 3 years. Give your answer correct to the nearest dollar.

WORKED Example 12

11 How long will it take $2000 to grow to $2500 when invested at 8% p.a. with interest compounded six-monthly?

12 Calculate the length of time that it will take:

- **a** $1000 to grow to $1100 at 10% p.a. with interest paid annually
- **b** $1000 to grow to $1500 at 7% p.a. with interest paid annually
- **c** $3000 to grow to $4000 at 6% p.a. with interest paid six-monthly
- **d** $9000 to grow to $10 000 at 8% p.a. with interest paid quarterly
- **e** $12 000 to grow to $17 500 at 10% p.a. with interest paid six-monthly.

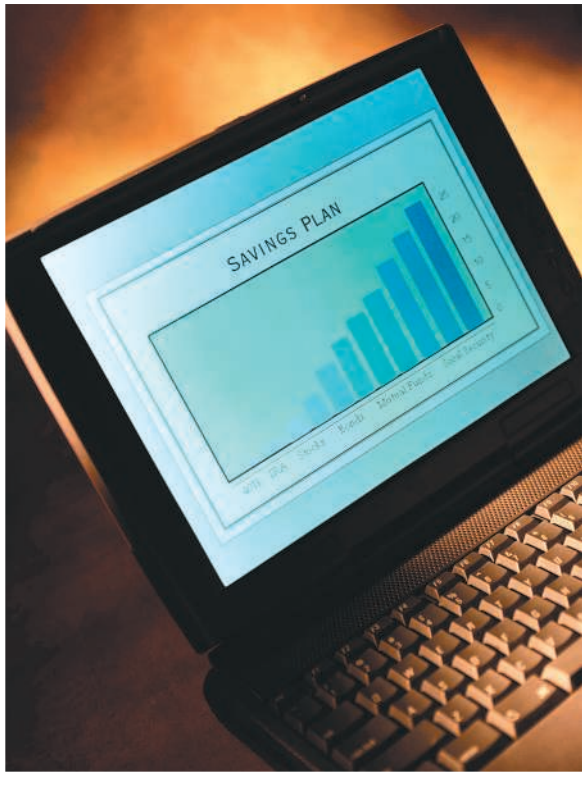
13 Calculate the interest rate required for $1000 to grow to $1300 in 2 years, if interest is compounded quarterly. (*Hint:* Find the *CVIF* required and use the table for 8 interest periods.)

14 Use the *CVIF* table to calculate the interest rate, to the nearest whole number, required for each of the following investments.

- **a** $1000 to grow to $1200 in 3 years with interest compounded annually
- **b** $2000 to grow to $2600 in 4 years with interest compounded six-monthly
- **c** $500 to grow to $650 in 1 year with interest compounded quarterly
- **d** $10 000 to grow to $20 000 in 8 years with interest compounded annually
- **e** $3500 to grow to $6000 in 5 years with interest compounded six-monthly

15 Bruce, Keith and Max each have $10 000 to invest over a 5-year term.

- **a** Bruce invests at 10% p.a. simple interest. Calculate the value of Bruce's investment at maturity.
- **b** Keith invests at 10% p.a. with interest compounded annually. Calculate the value of Keith's investment at maturity, using the *CVIF* table.
- **c** Max invests at 10% p.a. with interest compounded six-monthly. Calculate the value of Max's investment at maturity, using the *CVIF* table.
- **d** Calculate the total amount of interest each man received.
- **e** Write down the amount of interest each received as a percentage of their original investment.



Graphing compound interest functions

Earlier, we drew graphs of the simple interest earned by various simple interest investments and found that these graphs were linear. This occurred because the amount of interest earned in each interest period was the same.

With compound interest, the interest earned in each interest period increases, and so when we graph the future value of the investment, an exponential graph results. We can use the compounded values of $1 to complete tables that will then allow us to graph a compound interest function.

WORKED Example 13

Pierre invests $5000 at 5% p.a., with interest compounded annually.

a Use the table of compounded values to complete the table below to show the future value at the end of each year.

No. of years	1	2	3	4	5
Future value					

b Draw a graph of the future value of the investment against time.

THINK

a Use the table of compounded values of $1 to complete the table.

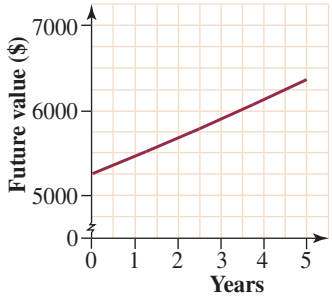
b Draw the graph, by drawing a smooth curve between the marked points.

WRITE/DRAW

a

No. of years	1	2	3	4	5
Future value	$5250	$5515	$5790	$6080	$6380

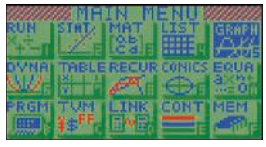
b



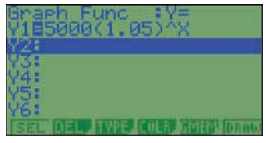
Graphics Calculator tip! Graphing a compound interest function

By using a graphics calculator, we can both calculate and draw a graph of the compound interest formula.

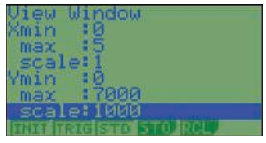
1. Write the compound interest formula. $A = P(1 + r)^n$
2. Substitute the known values of $P = \$5000$ and $r = 0.05$. Simplify the expression. $A = 5000(1 + 0.05)^n$ $= 5000(1.05)^n$
3. From the **MENU** select **GRAPH**.



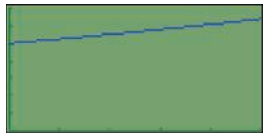
4. Delete any existing function and enter **Y1 = 5000(1.05)^X**.



5. To draw up the axes, press **SHIFT** **F3** for **V-Window** and enter the setting shown at right.



6. Press **EXE** to return to the previous screen and then press **F6** to **DRAW** the graph.



To graph the interest earned, the principal must be subtracted from the future value of the investment. As with simple interest, such graphs can be used to compare various investments.

WORKED Example 14

Amy is to invest $2000 at 5%, 6% or 7%, compounded annually.

a Copy and complete the table below to find the future value of each investment at the end of each year.

No. of years	1	2	3	4	5
Future value (5%)					
Future value (6%)					
Future value (7%)					

b Draw a graph that will allow the investments to be compared.

THINK

a Use the table of compounded values of $1 to complete the table.

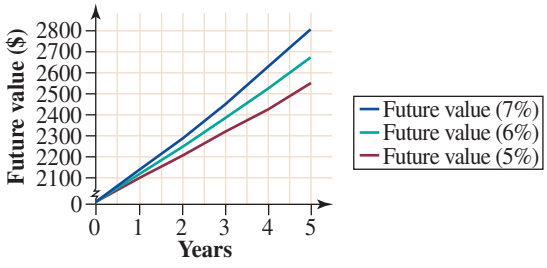
b Draw each graph by joining the points with a smooth curve.

WRITE/DRAW

a

No. of years	1	2	3	4	5
Future value (5%)	$2100	$2206	$2316	$2432	$2552
Future value (6%)	$2120	$2248	$2382	$2524	$2676
Future value (7%)	$2140	$2290	$2450	$2622	$2806

b



remember

1. A table that shows the compounded value of $1, at various interest rates for various compounding periods, can be used to calculate the future value of an investment.
2. Graphing the future value of a compound interest investment results in an exponential graph.
3. To graph the amount of compound interest paid, we need to subtract the principal from the future value.
4. By drawing the graphs of several investments on one set of axes, we can compare the investments.

EXERCISE 8E Graphing compound interest functions

WORKED Example 13

1 An amount of $8000 is invested at 5% p.a. with interest compounded annually.

a Copy and complete the table below, using the formula $A = P(1 + r)^n$, to calculate the future value of the investment at the end of each year.

No. of years	1	2	3	4	5
Future value					

b Draw a graph of the future value against the length of the investment.

2 An amount of $12 000 is to be invested at 8% p.a. with interest compounded annually.

a Copy and complete the table below to calculate the future value at the end of each year.

No. of years	1	2	3	4	5
Future value					

b Draw a graph of the interest earned against the length of the investment.

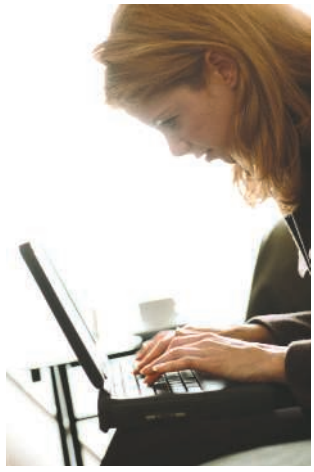
c Use your graph to find the future value of the investment after 10 years.

3 Draw a graph to represent the future value of the following investments against time.
- **a** $15 000 at 7% p.a. with interest compounded annually
- **b** $2000 at 10% p.a. with interest compounded annually

4 A graph is drawn to show the future value of an investment of $2000 at 6% p.a., with interest compounding six-monthly.
- **a** Complete the table below.

Years	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
FV										

- **b** Use the table to draw a graph.



5 An amount of $1200 is invested at 4% p.a. with interest compounding quarterly.
- **a** Graph the future value of the investment at the end of each year.
- **b** Graph the compound interest earned by the investment at the end of each year.

WORKED Example 14

6 James has $8000 to invest at either 4%, 6% or 8%, compounding annually.
- **a** Complete the table below to show the interest that he would earn over 5 years.

No. of years	1	2	3	4	5
Interest (4%)					
Interest (6%)					
Interest (8%)					

- **b** Show this information in graph form.

7 Petra has $4000 to invest at 6% p.a.
- **a** Complete the table below to show the future value of the investment at the end of each year, if interest is compounded annually, or six-monthly.

No. of years	1	2	3	4	5
Annually					
Six-monthly					

- **b** Show this information in graph form.

Share dividends

Investing money in banks and similar financial institutions is the most common type of investment, as it is safe and the return can be calculated in advance.

An alternative to investing in a bank is to purchase **shares**. Shares have a risk associated with them and there is no fixed return; however, they have the potential to return more money to the investor than through a bank.

When buying shares you are purchasing a share of the company. In other words, you become a part owner of that company. You can earn money from shares in two ways:

1. The profit made by a company will be paid to the company's owners (the shareholders). That part of the profit distributed to shareholders is called a **dividend**.
2. The value of shares changes daily. People invest in the share market with the expectation that the value of shares will rise and they can be sold at a profit. The risk is that the shares may fall in value.

Once or twice a year the directors of a company calculate the company's profit. A certain proportion of the profit may be spent on developing the company, the remainder being distributed to the shareholders as dividends.

A dividend is calculated by dividing the profit that is to be distributed by the number of shares in the company. The dividend is then declared on a per-share basis.

WORKED Example 15

A company has an after-tax profit of $34.2 million. There are 90 million shares in the company. What dividend will the company declare if all the profits are distributed to the shareholders?

THINK

1. The dividend is calculated by dividing the profit by the number of shares.
2. Give a written answer.

WRITE

$$\begin{aligned}\text{Dividend} &= \$34\,200\,000 \div 90\,000\,000 \\ &= \$0.38\end{aligned}$$

The dividend is 38c per share.

We can't accurately compare the values of investments from the dividend alone. We need to consider the money that was invested in order to earn that dividend. A 38c dividend paid by a company with a share value of $12.00 is a lower return than a company that pays a 15c dividend and has a share value of $2.50.

To compare the true return from any investment, we need to calculate that return as a percentage of the amount invested. For the income part of a share investment, this percentage is called the **dividend yield**. To calculate the dividend yield for any share, we calculate the dividend as a percentage of the share price.

WORKED Example 16

A company with a share price of $5.42 declares a dividend of 25c. Calculate the dividend yield, correct to 2 decimal places.

THINK

Write 0.25 (the dividend) over $5.42 (the share price) and multiply by 100%.

WRITE

$$\begin{aligned}\text{Dividend yield} &= \frac{0.25}{5.42} \times 100\% \\ &= 4.61\%\end{aligned}$$

remember

1. An investment in shares earns money through dividend payments and by increasing in value.
2. A dividend is a payment made to shareholders. It is calculated by dividing the profit to be distributed to shareholders by the total number of shares in the company.
3. To calculate the true worth of an investment, we calculate the dividend yield. The dividend yield is found by writing the dividend as a percentage of the share price.

EXERCISE 8F Share dividends

Unless stated otherwise, for the calculations in this exercise, assume that companies distribute all their profits as dividends.

SKILLSHEET 8.5 Finding a percentage of a quantity

WORKED Example 15

1 A company has issued 20 million shares and makes an after-tax profit of $5 million. Calculate the dividend to be declared by the company.

2 A company that has 2 million shares makes a profit of $3 million. Calculate the dividend that will be declared.

SKILLSHEET 8.6 Writing one quantity as a percentage of another

3 A company makes an after-tax profit of $150 000. If there are 2.5 million shares in the company, calculate the dividend that the company will declare.

4 A company with an after-tax profit of $1.2 million consists of 4.1 million shares. Calculate the dividend the company will declare, in cents, correct to 2 decimal places.

5 A company makes a before-tax (gross) profit of $3.4 million.
- **a** If the company is taxed at the rate of 36%, calculate the amount of tax it must pay.
- **b** What will be the after-tax profit of the company?
- **c** If there are 5 million shares in the company, calculate the dividend that the company will declare.

6 A company makes a gross profit of $14.5 million and there are 8 million shares in the company.
- **a** Calculate the after-tax profit if company tax is paid at the rate of 36%.
- **b** If $3.2 million is to be reinvested in the company, calculate the amount of money that is to be distributed to the shareholders.
- **c** Calculate the dividend that this company will declare.

7 A company declares a dividend of 78c. If there are 4.2 million shares in the company, calculate the after-tax profit of the company.

WORKED Example 16

8 A company with a share price of $10.50 declares a dividend of 48c per share. Calculate the dividend yield for this company.

9 Copy and complete the table below.

Dividend	Share price	Dividend yield
$0.56	$8.40	
$0.78	$7.40	
$1.20	$23.40	
$1.09	$15.76	
$0.04	$0.76	

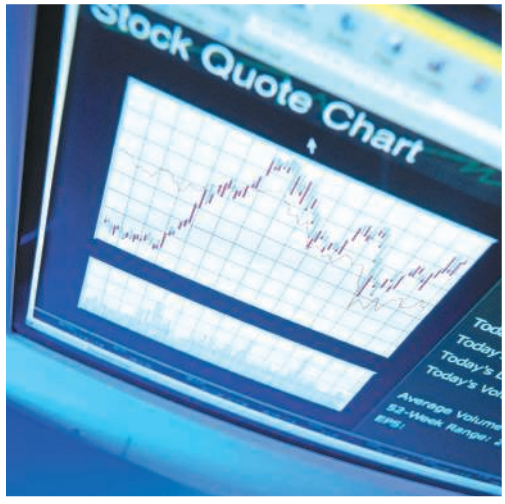
10 Hsiang purchased shares in a company for $3.78 per share. The company paid Hsiang a dividend of 11c per share. Calculate the dividend yield, correct to 2 decimal places.

11 **multiple choice**

Which of the following companies paid the highest dividend yield?

A Company A has a share value of $4.56 and pays a dividend of 35c/share.
B Company B has a share value of $6.30 and pays a dividend of 62c/share.
C Company C has a share value of $12.40 and pays a dividend of $1.10/share.
D Company D has a share value of 85c and pays a dividend of 7.65c/share.

12 George bought $5600 worth of shares in a company. The dividend yield for that company was 6.5%. Calculate the amount that George receives in dividends.



13 Andrea bought shares in a company for $11.50 each. The company paid a dividend of 76c/share.

a Calculate the dividend yield for this company.
b One year later the share value is $12.12. The company then has a dividend yield of 8.75%. Calculate the dividend per share.

14 A company's prospectus predicts that the dividend yield for the coming year will be 6.7%. Its share price is $21.50.

a Calculate the dividend paid if the dividend yield in the prospectus is paid.
b If there are 5.2 million shares in the company, calculate the after-tax profit of the company.

15 Janice buys shares in a company at $5.76. The company pays a dividend in July of 22.7c and a dividend in February of 26.4c. Calculate the dividend yield for the whole financial year (July to the following June).

16 The dividend paid by a company for the 2008–09 financial year was 5.6c/share, with a share price of $9.50.

a Calculate the dividend yield for 2008–09.
b In the 2009–10 financial year the share price rose by 12%. Calculate the share price for this year.
c In 2009–10 the dividend paid to shareholders increased by 15%. Calculate the dividend paid, in cents, correct to 1 decimal place.
d Calculate the dividend yield for 2009–10.

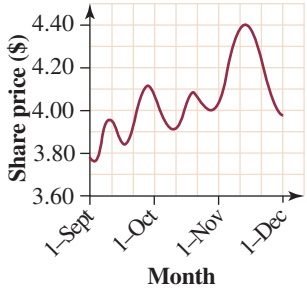
Graphing share performance

Because shares offer no guaranteed returns, we can only use the past performance of a share to try to predict its future performance. This is done by graphing the value of the share at regular intervals and then drawing a line of best fit to try to monitor the trend.

By continuing the line of best fit you can make a prediction for future share prices. This is called **extrapolating** information from the graph. **Interpolate** is the opposite of extrapolate and occurs when drawing a graph using data found at the end points.

WORKED Example 17

The graph shows the share price of a company over a 3-month period.



a On the graph draw a line of best fit.
b Use your line of best fit to estimate the share price after another three months.

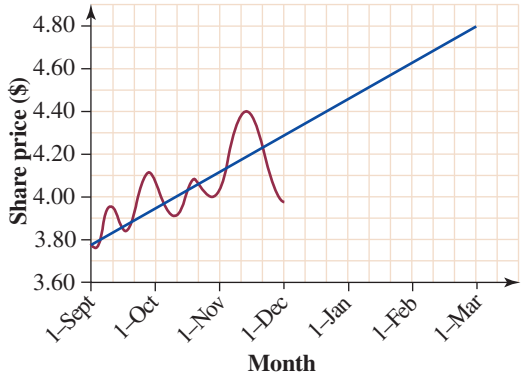
THINK

a Draw a line on the graph, which best fits between the points marked.

b Extend the line of best fit for three months and read the predicted share price.

WRITE

a



b The predicted share price is $4.80.

You should be able to produce your own graph to answer this type of question from a set of data that you have been given or have researched.

WORKED Example 18

Below is the share price of a company taken on the first day of the month for one year.

Month	Share price	Month	Share price
January	$10.34	July	$10.98
February	$10.54	August	$11.56
March	$10.65	September	$11.34
April	$10.89	October	$11.23
May	$10.72	November	$11.48
June	$11.10	December	$11.72

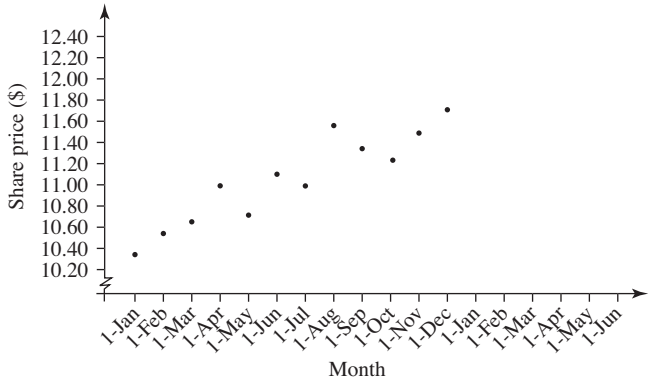
a On a set of axes plot the share price for each month and draw a line of best fit.
b Predict the share price in June of the following year.

THINK

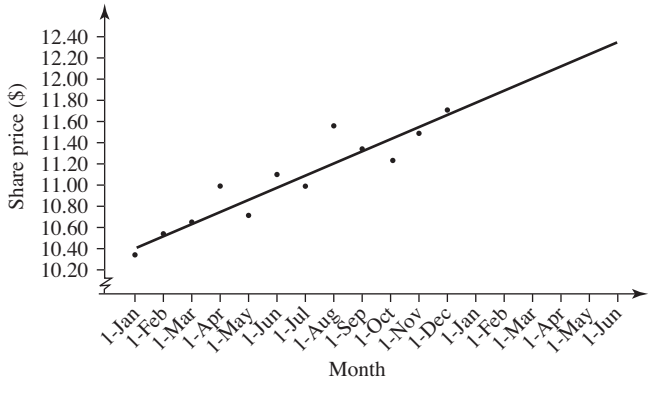
a 1 Draw up a set of axes and plot the data.

WRITE

a



2 Draw a straight line on the graph that best fits in with the marked points.



b 1 Extend the line of best fit for six months.
2 Predict the share price by reading from the line of best fit.

b

The predicted share price is $12.35.

remember

1. To try to predict possible future movement in share prices, we use the past performance of the share.
2. Graphing the past share price allows us to examine trends by drawing a line of best fit on the graph.
3. We can then use the line of best fit to predict the future price of a share.

EXERCISE 8G

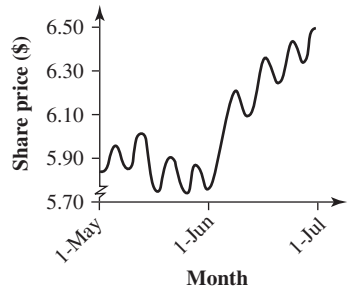
Graphing share performance

WORKED Example 17

1 The graph at right shows the movement in a share price over a 2-month period.

a Copy the graph into your book and on it draw a line of best fit.

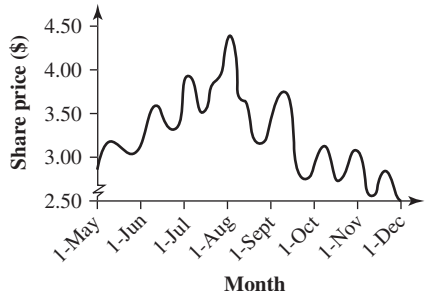
b Use your graph to predict the value of the share on 1 November.



2 The graph at right shows the movement in a share price over a 6-month period.

a Copy the graph into your book and on it draw a line of best fit.

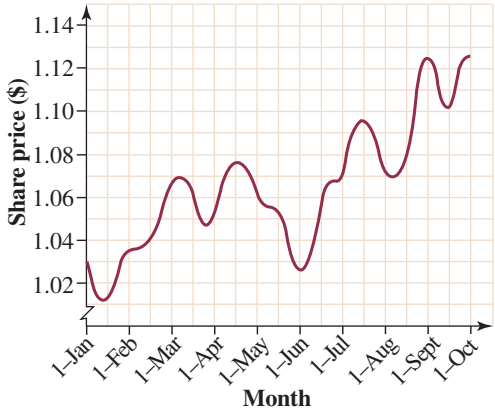
b Use your graph to predict the value of the share on 1 February.



3 The graph at right shows the movement in a share price over a 9-month period.

a Copy the graph into your book and on it draw a line of best fit.

b Use your graph to predict the value of the share after a further 12 months.



WORKED Example 18

4 The table below shows the share price of a large multinational company over a 12-month period.

Month	Share price	Month	Share price
January	$12.86	July	$13.45
February	$13.43	August	$13.86
March	$11.98	September	$14.40
April	$12.10	October	$13.65
May	$12.11	November	$13.20
June	$12.98	December	$12.86

a Plot the share prices on a set of axes and on your graph draw a line of best fit.
b Use your graph to predict the value of the share after a further 6 months.

5 The table below shows the share price of BigCorp Productions Ltd over a period of one year.

Month	Share price	Month	Share price
January	$12.40	July	$13.17
February	$12.82	August	$13.62
March	$12.67	September	$13.41
April	$13.05	October	$13.30
May	$13.06	November	$13.46
June	$12.89	December	$13.20

a Graph the share price for each month and show a line of best fit.
b Use your line of best fit to predict the share price in December of the next year.

Researching share prices

1. Choose three companies from the business section of the newspaper.
2. Determine the movement of each share over the past year using financial journals, the newspaper or the Internet.
3. Graph the information on the share price that you have found. Include the highest and lowest point of the share price over the past year.
4. On your graph, draw a line of best fit to find the overall trend in the movement of the share price.
5. Try to predict the share price in six months from now by extending the line of best fit.
6. Find the share price each week for six months and see if your line of best fit accurately predicts the share price.

Inflation and appreciation

One of the measures of how an economy is performing is the rate of **inflation**. Inflation is the rise in prices within an economy and is generally measured as a percentage. In Australia this percentage is called the Consumer Price Index (CPI). By looking at the inflation rate, we can estimate what the cost of various goods and services will be at some time in the future.

To estimate the future price of an item one year ahead, we increase the price of an item by the rate of inflation.

WORKED Example 19

The cost of a new car is $35 000. If the inflation rate is 5%, estimate the price of the car after one year.

THINK

Increase $35 000 by 5%.

WRITE

Future price = 105% of $35 000
= 105 ÷ 100 × $35 000
= $36 750

When calculating the future cost of an item several years ahead, the method of calculation is the same as for compound interest. This is because we are adding a percentage of the cost to the cost each year.

Remember the compound interest formula is $A = P(1 + r)^n$ and so in these examples P is the original price, r is the inflation rate expressed as a decimal and n is the number of years.

WORKED Example 20

The cost of a television set is $800. If the average inflation rate is 4%, estimate the cost of the television set after 5 years.

THINK

1. Write the values of P, r and n.
2. Write down the compound interest formula.
3. Substitute the values of P, r and n.
4. Calculate.

WRITE

$P = \$800$, $r = 0.04$, $n = 5$
$A = P(1 + r)^n$
$= \$800 \times (1.04)^5$
$= \$973.32$

A similar calculation can be made to anticipate the future value of collectable items, such as stamp collections and memorabilia from special occasions. This type of item increases in value over time if it becomes rare, and rises at a much greater rate than inflation. The amount by which an item grows in value over time is known as **appreciation**.

WORKED Example 21

Jenny purchases a rare stamp for $250. It is anticipated that the value of the stamp will rise by 20% per year. Calculate the value of the stamp after 10 years, correct to the nearest $10.

THINK	WRITE
1 Write the values of P, r and n.	$P = \$250$, $r = 0.2$, $n = 10$
2 Write down the compound interest formula.	$A = P(1 + r)^n$
3 Substitute the values of P, r and n.	$= \$250 \times (1.2)^{10}$
4 Calculate and round off to the nearest $10.	$= \$1550$

remember

1. Inflation is the measure of the rate at which prices increase.
2. The inflation rate is given as a percentage and is called the Consumer Price Index.
3. To estimate the cost of an item after one year, we increase the price by the percentage inflation rate.
4. To estimate the cost of an item after several years, we use the compound interest formula, using the inflation rate as the value of r.
5. Rare items such as collectibles and memorabilia increase in value as time goes on at a rate that is usually greater than inflation.

EXERCISE 8H Inflation and appreciation

WORKED Example 19

1 The cost of a motorcycle is $20 000. If the inflation rate is 4%, estimate the cost of the motorcycle after one year.

2 For each of the following, estimate the cost of the item after one year, with the given inflation rate.

- **a** An MP3 player costing $600 with an inflation rate of 3%
- **b** A toaster costing $45 with inflation at 7%

c A loaf of bread costing $1.80 with inflation at 6%
d An airline ticket costing $560 with inflation at 3.5%
e A washing machine costing $925 with inflation at 0.8%

3 An electric guitar is priced at $850 at the beginning of 2008.
a If the inflation rate is 3.3% p.a., estimate the cost of the guitar at the beginning of 2009.
b The government predicts inflation will fall to 2.7% in 2009. Estimate the cost of the guitar at the beginning of 2010.

4 When the Wilson family go shopping, the weekly basket of groceries costs $112.50. The inflation rate is predicted to be 4.8% for the next year. How much should the Wilson's budget per week be for groceries for the next year?

WORKED Example 20

5 The cost of a lawnmower is $550. If the average inflation rate is predicted to be 3%, estimate the cost of the lawnmower after 4 years.

6 The cost of a litre of milk is $1.70. If the inflation rate is an average 4%, estimate the cost of a litre of milk after 10 years.

7 A daily newspaper costs $1.00. With an average inflation rate of 3.4%, estimate the cost of a newspaper after 5 years (to the nearest 5c).

8 If a basket of groceries costs $98.50 in 2008, what would the estimated cost of the groceries be in 2015 if the average inflation rate for that period is 3.2%?

9 **multiple choice**

A bottle of soft drink costs $2.50. If the inflation rate is predicted to average 2% for the next five years, the cost of the soft drink in five years will be:
A $2.60 **B** $2.70 **C** $2.75 **D** $2.76

WORKED Example 21

10 Veronica bought a shirt signed by the Australian cricket team after it won the 2007 World Cup for $200. If the value of the shirt increases by 20% per annum for the next 5 years, calculate the value of the shirt (to the nearest $10).

11 Ken purchased a rare bottle of wine for $350. If the value of the wine is predicted to increase at 10% per annum, estimate the value of the wine in 20 years (to the nearest $10).

12 The 1968 Australian 2c piece is very rare. If a coin collector purchased one in 2006 for $400 and the value of the coin increases by 15% per year, calculate its value in 2019 (to the nearest $10).

WORKSHEET 8.2

summary

Simple interest

- Simple interest is interest paid where the interest is not added to the principal before the next interest calculation.
- It is calculated using the formula:

$$I = Prn \quad \text{FS}$$

where P is the initial quantity, r is the percentage interest rate per annum expressed as a decimal and n is the number of periods.
- It can be graphed as a linear function.

Compound interest

- Compound interest is the interest added to the principal before the next interest calculation is made.
- It can be calculated by using the formula:

$$A = P(1 + r)^n \quad \text{FS}$$

where A is the final balance, P is the initial quantity, r is the percentage interest rate per interest period expressed as a decimal and n is the number of compounding periods.
- The amount of compound interest paid is found by subtracting the principal from the future value of the investment.
- Compound interest can be calculated by using a table of compounded values of $1.
- When a compound interest function is graphed, it is an exponential function.

Shares

- When you buy shares you purchase a share in the company. There is no guaranteed return with shares, although there is a greater potential for profit than with investments such as banking and property, but with that comes a higher risk.
- Profit can be made from buying shares in two ways:
  (a) The value of the share could rise over time.
  (b) The company may pay a dividend to its shareholders. The dividend when written as a percentage of the share price is called the dividend yield.
- To try to predict the future movement in share prices, we can graph the past movement in the share price and draw a line of best fit on the graph. This line of best fit can be extrapolated to estimate the future price.

Inflation

- The price of goods and services rise from year to year. To predict the future price of an item, we can use the compound interest formula taking the rate of inflation to be r.
- The same method is used to predict the future value of collectibles and of memorabilia, which tend to rise at a rate greater than inflation.

CHAPTER review

8A 1 Calculate the simple interest earned on an investment of $5000 at 4% p.a. for 5 years.

8A 2 Calculate the simple interest earned on each of the following investments.
- a $3600 at 9% p.a. for 4 years
- b $23 500 at 6% p.a. for 2 years
- c $840 at 2.5% p.a. for 2 years
- d $1350 at 0.2% p.a. for 18 months
- e $45 820 at 4.75% p.a. for $3\frac{1}{2}$ years

8A 3 Dion invests $32 500 in a debenture paying 5.6% simple interest for 4 years.
- a Calculate the interest earned by Dion.
- b Calculate the total value of Dion's investment after 4 years.
- c If the debenture payed Dion in quarterly instalments, calculate the value of each interest payment.

8A 4 Bradley invests $15 000 for a period of 4 years. Calculate the simple interest rate, given that Bradley earned a total of $3900 interest.

8A 5 Kerry invests $23 500 at a simple interest rate of 4.6% p.a. If he earned $1351.25 in interest, calculate the length of time for which the money was invested.

8C 6 An amount of $7500 is to be invested at 6% p.a.
- a Copy and complete the table below to calculate the simple interest over 5 years.

No. of years	1	2	3	4	5
Interest					

- b Draw a graph of the interest earned against the length of the investment.
- c What is the gradient of the linear graph drawn?
- d Use your graph to find the amount of interest that would have been earned after 10 years.

8C 7 Vicky invests $2400 at 5% p.a. for 3 years with interest compounded annually. Calculate the compounded value of the investment at the end of the term.

8C 8 Barry has an investment with a present value of $4500. The investment is made at 6% p.a. with interest compounded six-monthly. Calculate the value of the investment in 4 years.

8C 9 Calculate the compounded value of each of the following investments.
- a $3000 at 7% p.a. for 4 years with interest compounded annually
- b $9400 at 10% p.a. for 3 years with interest compounded six-monthly
- c $11 400 at 8% p.a. for 3 years with interest compounded quarterly
- d $21 450 at 7.2% p.a. for 18 months with interest compounded six-monthly
- e $5000 at 2.6% p.a. for $2\frac{1}{2}$ years with interest compounded quarterly

8C 10 Dermott invested $11 500 at 3.2% p.a. for 2 years with interest compounded quarterly. Calculate the total amount of interest paid on this investment.

8C

11 Kim and Glenn each invest $7500 for a period of 5 years.
- **a** Kim invests her money at 9.9% p.a. with interest compounded annually. Calculate the compounded value of Kim's investment.
- **b** Glenn invests his money at 9.6% p.a. with interest compounded quarterly. Calculate the compounded value of Glenn's investment.
- **c** Explain why Glenn's investment has a greater compounded value than Kim's.

8C

12 Use the table of *CVIF* values on page 248 to calculate the compounded value of each of the following investments.
- **a** $6000 at 7% p.a. for 4 years with interest compounded annually
- **b** $7230 at 9% p.a. for 7 years with interest compounded annually
- **c** $3695 at 6% p.a. for 3 years with interest compounded six-monthly
- **d** $12 400 at 10% p.a. for 5 years with interest compounded six-monthly
- **e** $2400 at 4% p.a. for 2 years with interest compounded quarterly

8E

13 $20 000 is to be invested at 4% p.a. with interest compounded annually.
- **a** Copy and complete the table below, using the compound interest formula to calculate the future value at the end of each year.

No. of years	1	2	3	4	5
Future value					

- **b** Draw a graph of the interest earned against the length of the investment.
- **c** Use your graph to find the future value of the investment after 10 years.

8F

14 A company that has 10.9 million shares makes a profit of $21 million. If this entire amount is distributed among the shareholders, calculate the dividend that will be declared.

8F

15 A company that has an after-tax profit of $2.3 billion distributes this among its 156 million shares. Calculate the dividend that this company will declare.

8F

16 A company has a share price of $8.62. It declares a dividend of 45c per share. Calculate the dividend yield on this share.

8F

17 A company with a share price of 45c declares a dividend of 0.7c per share. Calculate the dividend yield on this investment.

8F

18 The dividend yield from a share valued at $19.48 is 4.2%. Calculate the dividend paid by the company, correct to the nearest cent.

8G

19 The table below shows the fluctuations in a share price over a period of 1 year.

Month	**Share price**	**Month**	**Share price**
January	$15.76	July	$16.60
February	$16.04	August	$16.77
March	$16.27	September	$16.51
April	$16.12	October	$16.71
May	$16.49	November	$16.69
June	$16.39	December	$16.98

- **a** On a set of axes plot the share price for each month.
- **b** Draw a line of best fit on your graph and use your line to predict the share price after a further year.

8H

8H

8H

20 A MP3 player is currently priced at $80. If the current inflation rate is 4.3%, estimate the price of the MP3 player after one year.

21 It is predicted that the average inflation rate for the next five years will be 3.7%. If a skateboard currently costs $125, estimate the cost of the skateboard after five years.

22 In 1979, Cherie bought a limited edition photograph autographed by Sir Donald Bradman for $120. If the photograph appreciates in value by 15% per annum, calculate the value of the photograph in 2009 (to the nearest $100).

Practice examination questions

1 multiple choice

The simple interest paid on $5600 at 5.6% for 3 years is:

A $940.80 **B** $994.46 **C** $6540.80 **D** $6594.47

2 multiple choice

The compound interest paid on $5600 at 5.6% for 3 years with interest compounded annually is:

A $940.80 **B** $994.46 **C** $6540.80 **D** $6594.47

3 multiple choice

A share is valued at $23.40. Greg buys 4000 shares and, at the end of the financial year, Greg receives a dividend of $4212. The dividend yield on Greg's investment is:

A 0.55% **B** 1.053% **C** 4.5% **D** 5.3%

4 multiple choice

In 2008, a basket of groceries costs $67.50. If the inflation rate is predicted to be 2.9% for the next year, by how much can we expect the cost of the basket of groceries to rise?

A $1.95 **B** $1.96 **C** $69.45 **D** $69.46

5 Jaclyn has $7500 saved for a holiday that she plans to take in two years time.

a If Jaclyn invests the money in a debenture that pays 4.2% p.a. simple interest, calculate the amount of money that Jaclyn will have after two years.

b An alternative investment for Jaclyn would be to invest her money at 4% p.a. for two years with interest compounding quarterly. Would this be a better investment? Explain your answer.

c Jaclyn finally decided to buy 1500 shares in a company at $5.00 each. For the past year the dividend yield for this company was 5.1%. Is this a safe investment for Jaclyn?

d After two years, the average dividend yield for this company was 4.8% p.a. of Jaclyn's initial investment and the shares were valued at $5.75 each. Calculate the total value of Jaclyn's investment.

e Calculate Jaclyn's profit as a percentage of her initial investment.

6 Frank has saved $30 000 to buy a new car. He decides to try to get another two years use out of his old car and in the meantime invest the money he has saved.

a If Frank invests the $30 000 at 3.5% p.a. for two years with interest compounded annually, calculate the money that Frank has at the end of the investment.

b Over the two years that Frank has invested his money, the inflation rate has averaged 4.2% p.a. Calculate the cost of the car at the end of this two years if the price rose at the same rate as inflation (to the nearest $100).

c How much more money does Frank now need to buy the new car?